FINAL: COMPUTER SCIENCE II

Date: **1st July 2016**

(1) (4+4+7=15 points) Write down the output of the following commands in octave.
  (a) x=[3 2 5]; A=x'*x; y=A(:,1); disp(min(y));
  (b) A=3*ones(3)+diag([-1, 0, 1]); B=A(1:2,1:2); fprintf('Determinant of B is %d \n',det(B));
  (c) x=[-pi:pi/4:pi]; plot(x,sin(x),"o-");

(2) (4+7+7+7=25 points) Describe what the following commands in octave do:
  (a) return
  (b) polyfit
  (c) chol
  (d) ode45

(3) (10 points) The function $y = \frac{x}{c_1 x + c_2 e^x}$ can be transformed into a linear relationship $z = c_1' w + c_2'$ with the change of variable $z = \frac{1}{y}$ and $w = \frac{e^x}{x}$. Write an "xlinxFit" function that calls linefit to fit data to $y = \frac{x}{c_1 x + c_2}$.

(4) (10 points) In the attached code for interpolation, identify the interpolation method and add comments in the code to explain the lines of the code which end with a % symbol. Also answer all the questions in mentioned there in the bold.

(5) (20 points) Let $\Phi_3(x) = \frac{1}{2}(5x^3 - 3x)$. Recall that $\Phi_3(x)$ is an orthogonal polynomial for the inner product $< f, g >= \int_{-1}^{1} f(t)g(t)dt$. Compute the weights for the Gaussian Quadrature method and use it to approximate $\int_0^2 e^{t^2} dt$.

(6) (20 points) Write down an octave function to find a solution to the differential equation

$$y' = e^{y-t} + cos(t),\ y(0) = 0$$

at t=2 using the stepsize h (which is a input variable for the function) following methods:
  (a) Euler method
  (b) Runge-Kutta method

```
function yhat = interpolate(x,f,fp,xhat)
% Name the interpolation method
%
% Synopsis: yhat = interpolate(x,f,fp,xhat)
%
% Input:    x     = vector of independent variable values
%           f, fp = vectors of f(x) and f'(x)
%           xhat = (scalar or vector) x values where interpolant is evaluated
%
% Output:   yhat = scalar or vector value of interpolant at
%                  x = xhat.  size(yhat) = size(xhat)

n = length(x);
if     length(f)~=n,    error('x and f are not compatible');
elseif length(fp)~=n,   error('x and fp are not compatible');   end

% --- Construct coefficients of the piecewise interpolants
x = x(:);  xhat = xhat(:);    %
f = f(:);  fp = fp(:);
dx = diff(x);                 %
divdif = diff(f)./dx;         %
a = f(1:n-1);
b = fp(1:n-1);
c = ( 3*divdif - 2*fp(1:n-1) - fp(2:n) ) ./dx;
d = ( fp(1:n-1) - 2*divdif + fp(2:n) )   ./dx.^2;

% What does the following four lines in the code do?
i = zeros(size(xhat));   %  i is index into x such that x(i) <= xhat <= x(i+1)
for m=1:length(xhat)     %  For vector xhat: x( i(m) ) <= xhat(m) <= x( i(m)+1 )
  i(m) = binSearch(x,xhat(m));
end

% --- Nested, vectorized evaluation of the piecewise polynomials
xx = xhat - x(i);
yhat = a(i) + xx.*(b(i) + xx.*(c(i) + xx.*d(i)) );
```